Mathematics 122 **Fall 2002**

Exam 5

December 10, 2002

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Name

Technology used:______________________ Directions:

Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

1. (3, 3, 4 points)

   (a) Give an example of a convergent infinite series.

   (b) Give an example of a divergent infinite sequence.

   (c) GIve an example of a monotone, non-increasing, unbounded sequence

2. (5, 5 points) Do any **two** of the following.

   (a) The Taylor Polynomial of degree 6 and with $c = 1$ for a function $f(x)$ is $P_6(x) = 12 - 4(x - 1) + 25(x - 1)^2 - 6(x - 1)^5 + 5(x - 1)^6$. What is $f^{(5)}(1)$?

   (b) Suppose we know that the infinite series $\sum_{n=1}^{\infty} c_n(x-1)^n$ converges at the value $x = 4$. Use the ideas of Radius of Convergence and Interval of Convergence to explain why we know the series converges at the value $x = -1$?

   (c) For this problem, assume the series $\sum_{k=1}^{\infty} \frac{(-1)^k}{\ln(k+1)}$ is a convergent alternating series. Use the error bound formula for alternating series to determine a value of n that guarantees that the n'th partial sum $S_n = \sum_{k=1}^{n} \frac{(-1)^k}{\ln(k+1)}$ of this series is accurate to within 10^{-1}. (This should strike you as 'slow convergence'.)

3. (10 points each) Determine if the following series are convergent or divergent. In part (d), use an appropriate 'Test' to determine if the convergence is absolute or conditional. Be sure to specify which tests you use.

   (a) $\sum_{k=5}^{\infty} \frac{1+k}{k}$

   (b) $\sum_{k=1}^{\infty} \frac{k!k!}{(2k)!}$

   (c) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\ln(k+1)}$

   (d) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^3}{4k^5-1}$

4. (10, 10 points) Given the series

$$\sum_{k=0}^{\infty} \frac{k(x-3)^k}{5^k}$$

   (a) Find the Radius of Convergence.

   (b) Find the Interval of Convergence.

5. (20 points) The formula for computing a Taylor Series for a function f at $x = c$ is

$$\sum_{k\,=\,0}^{\infty} \frac{f^{(k)}(c)}{k!}(x-c)^k$$

Use this formula to compute the Taylor Series with $c = 0$ (The Maclaurin Series) for the function $f(x) = \ln(1+x)$.